Sample Correlations

Classic Hypothesis Testing

$$\text{Let } \mathbf{x} = \begin{bmatrix} x_i \\ x_j \end{bmatrix} \sim \mathrm{N}_2\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right)$$

$$\text{Let } \rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \ \sigma_j}$$

Classic Hypothesis Testing

Then the test statistic

for testing the hypothesis

$H_0 : \rho_{ij} = 0$

$H_1 : \rho_{ij} \neq 0$

is

$$t_{ij} = \sqrt{N-2}\,\frac{r_{ij}}{\sqrt{1-r_{ij}^2}} \sim t_{N-2}$$

Confidence Intervals

- Confidence Intervals for ρ_{ij} can be constructed using
  - Fisher's z-transformation
    - Derived as the variance stabilizing transformation
  - Reuben's approximation

Fisher's z-transformation

$$\text{Let } z = \frac{1}{2}\ln\left(\frac{1+r}{1-r}\right) = \tanh^{-1}(r)$$

$$\text{Then } z \sim \text{N}\left[\left\{\frac{1}{2}\ln\left(\frac{1+\rho}{1-\rho}\right) - \frac{\rho}{2(N-1)}\right\}, \frac{1}{N-3}\right]$$

Bias Adjustment

$\frac{\rho}{2(N-1)}$ is referred to as the bias adjustment associated with the distribution of Fisher's z transformation

Fisher’s z-transformation
CI without bias adjustment

A 100(1-α)% CI for ρ is given by

$$\tanh\left(\tanh^{-1}(r) \pm \frac{z_{\alpha/2}}{\sqrt{N-3}} \right)$$

Fisher’s z-transformation
CI with bias adjustment

A 100(1-α)% CI for ρ is given by

$$\tanh\left(\tanh^{-1}(r) - \frac{r}{2\left(N-1\right)} \pm \frac{z_{\alpha/2}}{\sqrt{N-3}} \right)$$

Reuben's Approximation

A 100(1-α)% CI for ρ is given by

$$\left(\frac{y_1}{\sqrt{\left(1+y_1^2\right)}}, \frac{y_2}{\sqrt{\left(1+y_2^2\right)}} \right)$$

Reuben's Approximation

where

y_1 and y_2 are the roots of

$$ay^2 - 2by + c = 0$$

Reuben's Approximation

where

$$a = 2N - 3 - z_{\alpha/2}^2$$

$$b = \frac{r}{\sqrt{1-r^2}}\sqrt{(2N-3)(2N-5)}$$

$$c = \left(2N - 5 - z_{\alpha/2}^2\right)\frac{r^2}{1-r^2} - 2\,z_{\alpha/2}^2$$