The Analytic Use of Two-Level Factorials in Incomplete Blocks to Examine the Stability of Factor Effects

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Abstract

When used in industry, incomplete block designs for 2^k factorials should allow the experimenter to analyze the consistency of factor effects across blocks. This paper presents a summary of designs appropriate for this type of investigation and describes the correct analysis of these designs.

Introduction

The traditional purpose of the randomized block design is to remove the effects that distinguish blocks in order to make the presence of other effects more apparent. However, in industrial experimentation blocking can also be used to determine whether the effects noted in an initial experiment are repeatable across blocks or are influenced by other factors that change from block to block. In other words, blocking is used to examine the stability of factor effects across different processing conditions. Sanders, Leitnaker, and McLean (1) discuss the use of complete block designs for this purpose. However, for situations in which we want to study a large number of factors, it will not generally be possible to conduct experiments with complete blocks. This paper explores the use of incomplete block designs for 2^k and 2^{k-p} fractional factorials in situations where it is necessary to evaluate whether factors interact with blocks.

The value of running industrial experiments in blocks is the ability to explicitly examine the stability of factor effects across time and conditions. For example, many processes are affected by the changing characteristics of incoming raw materials and the processes must be managed with this in mind. In such situations, experiments performed only on, say, one species of wood or one lot of plastic pellets would not provide information about whether effects of the experimental factors were consistent across differing raw materials. In other processes, differentials between set-ups, maybe performed daily, are likely to result in anticipated but poorly understood sources of variation. Here again, designed experiments run in blocks across several set-ups will enable us to determine if factor effects are consistent across set-ups. Similar situations abound in industrial processes. We may have anticipated but unmanaged changes in process characteristics due to shift differences, time-of-day influences, or even simply a slow drift over time. Certainly good process management would mean that we would try to remove such sources of variation. However, it is still often necessary to optimize a process in light of such processing conditions. Fractional factorials run in blocks provide a technique for not only assessing the effects of suspected influential factors but also evaluating whether these effects persist across changing conditions.

Literature Review

Fractional factorials have proved extremely valuable in industrial experimentation. The need to run experiments in as efficient a manner as possible, and yet still obtain reliable results, has dictated the use of fractional factorials. Box, Hunter, and Hunter (2) is a classic text for instructing engineers in the use of fractional factorials that allow the engineer to acquire useful process information for directing process improvement. An additional constraint often found in industrial experimentation is that it is often not possible to complete many runs within a short time frame. For this reason the use of blocks in experimentation is necessary. Young (3) has also written about the need to use blocking in industrial experimentation in order to increase the precision of experimental results.

It is a valuable historical note to return to some of the early work done with factorial experiments in blocks. The foundational work by Yates (4) on factorial experiments addressed the issue of "differential responses in different blocks" (i.e. blocks*factor interactions) in agricultural experiments. Yates' primary concern was that block*factor interactions would invalidate the pooled error estimators commonly used. However, in the case of industrial experimentation, the main concern with differential responses in different blocks is that, if present, the results of the experiment are not consistent across conditions, and hence recommendations for process operation can not be made without either an understanding of (i.) why these differential responses are occurring or (ii.) whether the nature of the differential responses will still allow for process optimization. Kempthorne (5) also discusses the assumption of differential responses since "the interpretation of many factorial designs involving a high degree of confounding or fractional replication depends on the assumption that block-treatment interactions are negligible." His conclusion, based on the analysis of 128 agricultural experiments, is that this assumption is reasonable. However, we believe that this conclusion is not transferable to industrial experiments. Although it has become common practice since Yates' and Kempthorne's work to assume no block-factor interactions, for reasons noted previously, this should not be done in most industrial experiments. In fact, the investigation of block-factor interactions is often one of the objectives of an industrial experiment.

In this paper, we provide some fractional factorial designs that can be used not only to run fractional factorials in blocks, but also to check for the existence of differential responses across the (generally time-ordered) blocks of the experiment. Next we describe a method for analyzing this type of experiment. Finally, we provide an example to concretely illustrate these ideas.

Selecting an Incomplete Block Factorial Design

The selection of an incomplete block design will, of course, depend on

- the number of factors to be studied,
- the size of the block (generally governed by the length of time needed to complete a run),
- and the number of blocks of runs that can be performed (governed by the amount of processing time to be devoted to an experiment).

In addition, the choice of a design should also recognize the need to understand block-factor interactions. There are a number of references by which one might obtain an incomplete block design. Box, Hunter, and Hunter (2), Montgomery (6), Sun, Wu, and Chen (7), and many software packages provide such designs. However, the information provided with these designs does not include the confounding structure of the block-factor interactions. Since we recommend that the analysis of these block*factor interactions is a critical component of the analysis of incomplete block designs, this information is vital. More importantly, if designs are chosen without regard to the confounding of block*factor interactions, the chosen design may not be optimal for evaluating these interactions.

Table 1 is a list of incomplete block designs for experiments that have from three to eight factors. A listing of these designs and their block*factor interaction structure is provided in the Appendix. Two principles have guided the selection of the designs in the Appendix. First and foremost is the ability to estimate the block*factor interactions in conjunction with an examination of other two-factor interactions. And secondly, the designs chosen have blocks that are Resolution III or higher. Thus, if the experimenter is unable to complete all blocks, it will still be possible to estimate main effects for the block or blocks that were run.

Table 1. Recommended Designs for k = 3, …, 8 Factors

No. of factors	Block Size	No. of blocks	Resolution of blocks
3	4	4	III
4	8	2	IV
5	8	4	III
5	16	2	V
6	16	4	IV
6	8	8	III
7	16	4	IV
7	8	8	III
8	16	4	IV
8	16	8	IV

For example, consider the design for 6 factors run in 8 blocks of size 8. This design has blocks created by confounding:

Blk=CDEF   Blk=ABEF   Blk=BDF.

As a result, the seven degrees of freedom for blocks also include the following confounded interactions:

Blk=ABCD   Blk=BCE   Blk=ADE   Blk=ACF.

There are 2^6=64 runs in this design, the same number as a full factorial with no blocking. Therefore the block*factor interactions, which require 7 degrees of freedom to estimate, must be confounded with seven other effects. These effects, as determined from the above aliasing structure, are provided in Table 2 below.

Table 2: Block-Factor Confounding for 6 Factors in 8 Blocks of Size 8

Blk*A	Blk*B	Blk*C	Blk*D	Blk*E	Blk*F
DE	CE	BE	BF	BC	BD
CF	DF	AF	AE	AD	AC
BCD	ACD	ABD	CEF	CDF	CDE
BEF	AEF	DEF	ABC	ABF	ABE
ABCE	ABDE	BCDF	BCDE	BDEF	BCEF
ABDF	ABCF	ACDE	ACDF	ACEF	ADEF
ACDEF	BCDEF	ABCEF	ABDEF	ABCDE	ABCDF

An examination of the above confounding structure shows that the block*factor interactions are each confounded with two 2-factor interactions as well as with five other higher order interactions. The analyses of experimental data will exploit this structure. The two-factor interactions will be estimated separately. The remaining five degrees of freedom left for estimating block-factor interactions will be assumed to provide information about possible differential responses of main effects across blocks. In other words, the existence of three-factor or higher order interactions is assumed negligible.

Now consider the design for seven factors run in 4 blocks of size sixteen. Here there are a total of 2^{7-1}=64 runs, so specifying a design will entail not only describing the confounding of blocks with effects, but also specifying a treatment defining contrast for the one-half fraction. In this case our choice of an optimal design will be different than if we did not intend to estimate block*factor interactions. The optimal choices suggested by JMP software, by Box, Hunter, and Hunter (2), and Sun, Wu, and Chen (7) are resolution V designs (designs which do not confound two-way interactions with each other) but the individual blocks are Resolution III. However, by choosing a resolution IV design for this case, our design also has blocks that are resolution IV. The implications of this choice are two-fold. First, if only the first block is run, we can still estimate main effects and some two-factor interactions for the data from that block. Also, block*factor interactions are not confounded with any two-factor interactions. Since in this case there will only be three degrees of freedom available to examine block*factor interactions, we feel it is important to be able to use all of these to understand the consistency of the factor effects across blocks.

Analyzing the Incomplete Block Factorial Experiment

The six-factor experiment, as with all of the designs listed in the Appendix, should be analyzed by the sequence of steps listed below.

Step 1: The main effects, then all two-factor interactions, then all block-factor interactions should be included in a model statement in that order. It is necessary that the effects be included in the model statement in this order, as the Type 1 (also called sequential) sums of squares should be used to analyze the experimental data. Of course, if no two factor interactions are confounded with block*treatment effects, the Type I sum of squares will be the same as the Type III.

Step 2: Examine the Type 1 mean squares for the block-factor interactions to see if any stand out. Because the Type 1 mean squares are being used, a large two-factor interaction will not inflate the magnitude of the block*factor interactions. If any of these block*treatment effects are large, then the experimenter must address the reason why the factor(s) involved in this large interaction do not have a consistent effect across the blocks in the experiment. In other words, the large mean square indicates that the effect of this factor is not consistent across the experiment. So any conclusions about this factor must take into account the fact that the overall main effect observed will not be observed across the changing conditions of the process under study. A plot of the main effects by block will help the experimenter understand the changing nature of the main effects across blocks.

Also, if a particular block-factor interaction is large, then any large sum of squares for a two-factor interaction confounded with this block-factor interaction should be treated with suspicion. It is possible that the apparent two-factor interaction is actually due to the presence of the block-factor interaction.

Step 3: If none of the block-factor interactions are large, then these effects may be dropped from the model statement. Analysis of the experimental data can then proceed as is typical of a factorial experiment.

To illustrate the steps in the analysis, a set of experimental data has been simulated. The simulation used the following equation to generate the data for the experiment:

$$Y_{ijkl} = 25 + 5A_i + 5B_j + 2.5AB_{ij} + 3C_k I(Blk<4) + \varepsilon_{l(ijk)}$$

Following Step 1, the ANOVA Table below was generated in JMP from the simulated data.

Summary of Fit

RSquare	0.998582
RSquare Adj	0.982137
Root Mean Square Error	1.062552
Mean of Response	25.04158
Observations (or Sum Wgts)	64

Sequential (Type 1) Tests

Source	Nparm	DF	Seq SS	F Ratio	Prob>F
Block	7	7	4.2511	0.5379	0.7799
A	1	1	1651.7336	1462.985	0.0000
B	1	1	1649.6909	1461.176	0.0000
A*B	1	1	408.0355	361.4081	0.0000
C	1	1	79.6562	70.5536	0.0004
A*C	1	1	0.1373	0.1216	0.7415
B*C	1	1	0.6595	0.5842	0.4792
D	1	1	0.3468	0.3071	0.6033
A*D	1	1	0.9407	0.8332	0.4032
B*D	1	1	0.2193	0.1943	0.6778
C*D	1	1	0.8240	0.7298	0.4320
E	1	1	1.4858	1.3160	0.3032
A*E	1	1	1.7208	1.5242	0.2718
B*E	1	1	3.9900	3.5341	0.1189
C*E	1	1	0.1198	0.1061	0.7578
D*E	1	1	0.3748	0.3320	0.5895
F	1	1	1.5263	1.3519	0.2974
A*F	1	1	8.2861	7.3392	0.0423
B*F	1	1	0.2332	0.2066	0.6685
C*F	1	1	2.6059	2.3082	0.1892
D*F	1	1	0.3974	0.3520	0.5788
E*F	1	1	0.0843	0.0747	0.7956
Block*A	7	5	1.1568	0.2049	0.9466

Block*B	7	5	4.6093	0.8165	0.5853
Block*C	7	5	134.1927	23.7716	0.0017
Block*D	7	5	2.1088	0.3736	0.8482
Block*E	7	5	5.9781	1.0590	0.4757
Block*F	7	5	10.7861	1.9107	0.2472

Whole-Model Test Analysis of Variance

Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**
Model	58	3976.1512	68.5543	60.7204
Error	5	5.6451	1.1290	**Prob>F**
C Total	63	3981.7963		0.0001

The Type I Mean Square for Block*C (134.1927/5) is, as expected from the simulated data equation, large. When such a situation arises in practice, we should question the presence of an A*F interaction and construct plots to graphically depict the behavior of the factor C across the different blocks in the experiment.

Example from Cochran and Cox

The example provided in this section appeared in Cochran and Cox (8, section 6.14.) This example provides both a simple extension to the ideas of blocking and fractional factorials we have previously discussed as well as an illustration of the method and value of analyzing such experimental data as proposed earlier.

The field experiment, conducted at Rothamsted, investigated the effects of four nutrient factors on the yield of beans. These four factors were:

D – dung, K – potash, N - nitrochalk, and P - phosphate.

Thirty-two runs of the experiment were completed by arranging 32 plots in four blocks of size eight. For studying four factors, Table 1 suggests using two blocks of size eight. The experiment in

Cochran and Cox is a simple extension of this design; two replications of this basic design were performed. And our recommendation is that the analysis of this experiment proceed as described previously.

Before performing this analysis, it is instructive to examine the analysis by Cochran and Cox. They assumed no treatment*block interactions and found two significant effects, the nitrochalk main effect and a dung*phosphate interaction. However, including the treatment*block interactions in the analysis calls these results into question. The JMP output for our recommended analysis is provided below.

Summary of Fit

RSquare	0.970459
RSquare Adj	0.84737
Root Mean Square Error	2.508319
Mean of Response	46.9375
Observations (or Sum Wgts)	32

Sequential (Type 1) Tests

Source	**Nparm**	**DF**	**Seq SS**	**F Ratio**	**Prob>F**
Blk	3	3	126.37500	6.6954	0.0242
D	1	1	2.00000	0.3179	0.5933
N	1	1	325.12500	51.6755	0.0004
D*N	1	1	32.00000	5.0861	0.0650
P	1	1	6.12500	0.9735	0.3619
D*P	1	1	242.00000	38.4636	0.0008
N*P	1	1	78.12500	12.4172	0.0125
K	1	1	4.50000	0.7152	0.4301
D*K	1	1	6.12500	0.9735	0.3619
N*K	1	1	32.00000	5.0861	0.0650
P*K	1	1	24.50000	3.8940	0.0959
Blk*D	3	3	130.50000	6.9139	0.0225
Blk*N	3	3	141.37500	7.4901	0.0188
Blk*P	3	3	19.37500	1.0265	0.4449
Blk*K	3	3	70.00000	3.7086	0.0807

Whole-Model Test Analysis of Variance

Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**
Model	25	1240.1250	49.6050	7.8842
Error	6	37.7500	6.2917	**Prob>F**
C Total	31	1277.8750		0.0081

The initial evaluation of this type of experiment must first evaluate the effect, if any, of the block*treatment interactions. An examination of the above output shows that both D (dung) and N (nitrochalk) have effects which are not the same across the four blocks since the Blk*D and Blk*N effects are large. An examination of these effects across the blocks is indicated. The plot in Figure 1 provides a visual explanation of these significant effects.

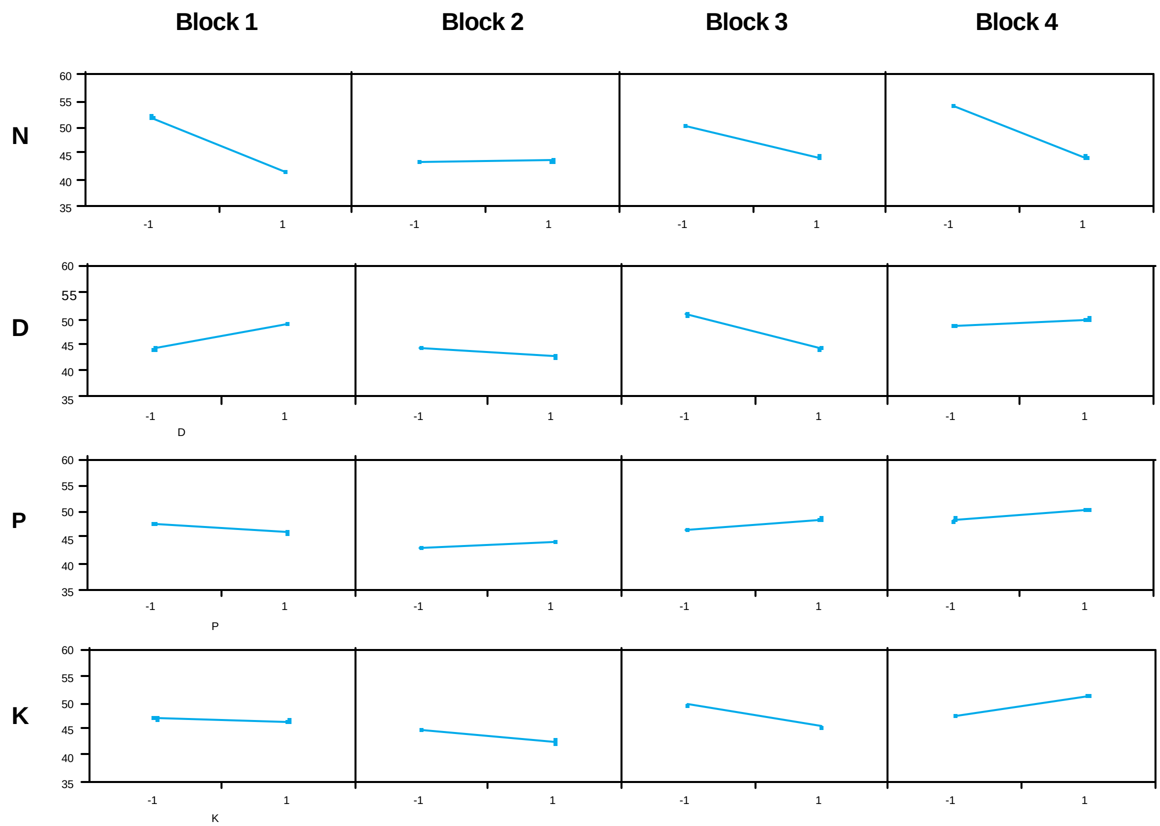

Figure 1: Plot of main effects by block for Cochran and Cox (8) data

The plots in Figure 1 indicate that only in two blocks, one and four, is there a strong nitrochalk effect. The plots of the main effect for dung are even more interesting. In block one there is an increase in yield with increasing dung, whereas in block three there is a marked decrease. In both cases, one could not conclude there is an increasing or decreasing effect in yield due to dung or to nitrochalk across all blocks.

Summary

In this paper we have proposed a different approach to analyzing the results of fractional factorials run in blocks. Particularly in industrial experimentation, the possibility and importance of block*treatment interactions demands that these interactions be properly analyzed. Since the analysis of block*treatment interactions has not typically been stressed, experimental designs for fractional factorials in blocks have often not been constructed with this analysis need in mind. The designs provided in this paper provide both optimal initial blocks and multi-block designs suitable for assessing stability of main effects across blocks.

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Key Words

Block-treatment interactions, fractional factorial

Biographical Sketches

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APPENDIX

Design 1. k = 3, n = 16 in 4 blocks of size 4

Design 2 k = 4, n = 16 in 2 blocks of size 8

Design 3 k = 5, n = 32 in 4 blocks of size 8

Design 4 k = 5, n = 32 in 2 blocks of size 16

Design 5 k = 6, n = 64 in 4 blocks of size 16

Design 6 k = 6, n = 64 in 8 blocks of size 8

Design 7 k = 7, n = 64 in 4 blocks of size 16

Design 8 k = 7, n = 64 in 8 blocks of size 8

Design 9 k = 8, n = 64 in 4 blocks of size 16

Design 10 k = 8, n = 128 in 8 blocks of size 16

Design 1: 3 Factors in 4 Blocks of Size 4

Blk	A	B	C
1	-1	-1	1
1	-1	1	-1
1	1	-1	-1
1	1	1	1
2	-1	-1	-1
2	-1	1	1
2	1	-1	1
2	1	1	-1
3	-1	-1	-1
3	-1	1	1
3	1	-1	1
3	1	1	-1
4	-1	-1	1
4	-1	1	-1
4	1	-1	-1
4	1	1	1

Alias Structure: Blk = ABC

Block*Factor Confounding Structure:

Blk*A	Blk*B	Blk*C
BC	AC	AB

Design 2: 4 Factors in 2 Blocks of Size 8

Blk	A	B	C	D
1	-1	-1	-1	1
1	-1	-1	1	-1
1	-1	1	-1	-1
1	-1	1	1	1
1	1	-1	-1	-1
1	1	-1	1	1
1	1	1	-1	1
1	1	1	1	-1
2	-1	-1	-1	-1
2	-1	-1	1	1
2	-1	1	-1	1
2	-1	1	1	-1
2	1	-1	-1	1
2	1	-1	1	-1
2	1	1	-1	-1
2	1	1	1	1

Alias Structure: Blk = ABCD

Block*Factor Confounding Structure:

Blk*A	Blk*B	Blk*C	Blk*D

Note: No two factor interactions are confounded with Blk*X

Design 3: 5 Factors in 4 Blocks of Size 8

Blk	A	B	C	D	E
1	-1	-1	1	-1	-1
1	-1	-1	1	1	1
1	-1	1	-1	-1	-1
1	-1	1	-1	1	1
1	1	-1	-1	-1	1
1	1	-1	-1	1	-1
1	1	1	1	-1	1
1	1	1	1	1	-1
2	-1	-1	-1	-1	-1
2	-1	-1	-1	1	1
2	-1	1	1	-1	-1
2	-1	1	1	1	1
2	1	-1	1	-1	1
2	1	-1	1	1	-1
2	1	1	-1	-1	1
2	1	1	-1	1	-1
3	-1	-1	-1	-1	1
3	-1	-1	-1	1	-1
3	-1	1	1	-1	1
3	-1	1	1	1	-1
3	1	-1	1	-1	-1
3	1	-1	1	1	1
3	1	1	-1	-1	-1
3	1	1	-1	1	1
4	-1	-1	1	-1	1
4	-1	-1	1	1	-1
4	-1	1	-1	-1	1
4	-1	1	-1	1	-1
4	1	-1	-1	-1	-1
4	1	-1	-1	1	1
4	1	1	1	-1	-1
4	1	1	1	1	1

Alias Structure: Blk = BCDE Blk = ADE

Block*Factor Confounding Structure:

Blk*A	Blk*B	Blk*C	Blk*D	Blk*E
DE	AC	AB	AE	AD
BC				

Design 4: 5 Factors in 2 Blocks of Size 16

Blk	A	B	C	D	E
1	-1	-1	-1	-1	-1
1	-1	-1	-1	1	1
1	-1	-1	1	-1	1
1	-1	-1	1	1	-1
1	-1	1	-1	-1	1
1	-1	1	-1	1	-1
1	-1	1	1	-1	-1
1	-1	1	1	1	1
1	1	-1	-1	-1	1
1	1	-1	-1	1	-1
1	1	-1	1	-1	-1
1	1	-1	1	1	1
1	1	1	-1	-1	-1
1	1	1	-1	1	1
1	1	1	1	-1	1
1	1	1	1	1	-1
2	-1	-1	-1	-1	1
2	-1	-1	-1	1	-1
2	-1	-1	1	-1	-1
2	-1	-1	1	1	1
2	-1	1	-1	-1	-1
2	-1	1	-1	1	1
2	-1	1	1	-1	1
2	-1	1	1	1	-1
2	1	-1	-1	-1	-1
2	1	-1	-1	1	1
2	1	-1	1	-1	1
2	1	-1	1	1	-1
2	1	1	-1	-1	1
2	1	1	-1	1	-1
2	1	1	1	-1	-1
2	1	1	1	1	1

Alias Structure: Blk = ABCDE

Block*Factor Confounding Structure:

Blk*A	Blk*B	Blk*C	Blk*D	Blk*E

Note: No two factor interactions are confounded with Blk*X

Design 5: 6 Factors in 4 Blocks of Size 16

Blk	A	B	C	D	E	F	Blk	A	B	C	D	E	F
1	-1	-1	-1	-1	-1	1	3	-1	-1	-1	1	-1	-1
1	-1	-1	-1	-1	1	-1	3	-1	-1	-1	1	1	1
1	-1	-1	1	1	-1	1	3	-1	-1	1	-1	-1	-1
1	-1	-1	1	1	1	-1	3	-1	-1	1	-1	1	1
1	-1	1	-1	1	-1	-1	3	-1	1	-1	-1	-1	1
1	-1	1	-1	1	1	1	3	-1	1	-1	-1	1	-1
1	-1	1	1	-1	-1	-1	3	-1	1	1	1	-1	1
1	-1	1	1	-1	1	1	3	-1	1	1	1	1	-1
1	1	-1	-1	1	-1	-1	3	1	-1	-1	-1	-1	1
1	1	-1	-1	1	1	1	3	1	-1	-1	-1	1	-1
1	1	-1	1	-1	-1	-1	3	1	-1	1	1	-1	1
1	1	-1	1	-1	1	1	3	1	-1	1	1	1	-1
1	1	1	-1	-1	-1	1	3	1	1	-1	1	-1	-1
1	1	1	-1	-1	1	-1	3	1	1	-1	1	1	1
1	1	1	1	1	-1	1	3	1	1	1	-1	-1	-1
1	1	1	1	1	1	-1	3	1	1	1	-1	1	1
2	-1	-1	-1	1	-1	1	4	-1	-1	-1	-1	-1	-1
2	-1	-1	-1	1	1	-1	4	-1	-1	-1	-1	1	1
2	-1	-1	1	-1	-1	1	4	-1	-1	1	1	-1	-1
2	-1	-1	1	-1	1	-1	4	-1	-1	1	1	1	1
2	-1	1	-1	-1	-1	-1	4	-1	1	-1	1	-1	1
2	-1	1	-1	-1	1	1	4	-1	1	-1	1	1	-1
2	-1	1	1	1	-1	-1	4	-1	1	1	-1	-1	1
2	-1	1	1	1	1	1	4	-1	1	1	-1	1	-1
2	1	-1	-1	-1	-1	-1	4	1	-1	-1	1	-1	1
2	1	-1	-1	-1	1	1	4	1	-1	-1	1	1	-1
2	1	-1	1	1	-1	-1	4	1	-1	1	-1	-1	1
2	1	-1	1	1	1	1	4	1	-1	1	-1	1	-1
2	1	1	-1	1	-1	1	4	1	1	-1	-1	-1	-1
2	1	1	-1	1	1	-1	4	1	1	-1	-1	1	1
2	1	1	1	-1	-1	1	4	1	1	1	1	-1	-1
2	1	1	1	-1	1	-1	4	1	1	1	1	1	1

Alias Structure: Blk=CDEF Blk=ABEF

Block*Factor Confounding Structure:

Blk*A	Blk*B	Blk*C	Blk*D	Blk*E	Blk*F

Note: No two factor interactions are confounded with Blk*X

Design 6: 6 Factors in 8 Blocks of Size 8

Blk	A	B	C	D	E	F	Blk	A	B	C	D	E	F
1	-1	-1	-1	-1	1	-1	5	-1	-1	-1	-1	-1	1
1	-1	-1	1	1	-1	1	5	-1	-1	1	1	1	-1
1	-1	1	-1	1	-1	-1	5	-1	1	-1	1	1	1
1	-1	1	1	-1	1	1	5	-1	1	1	-1	-1	-1
1	1	-1	-1	1	1	1	5	1	-1	-1	1	-1	-1
1	1	-1	1	-1	-1	-1	5	1	-1	1	-1	1	1
1	1	1	-1	-1	-1	1	5	1	1	-1	-1	1	-1
1	1	1	1	1	1	-1	5	1	1	1	1	-1	1
2	-1	-1	-1	1	-1	1	6	-1	-1	-1	1	1	-1
2	-1	-1	1	-1	1	-1	6	-1	-1	1	-1	-1	1
2	-1	1	-1	-1	1	1	6	-1	1	-1	-1	-1	-1
2	-1	1	1	1	-1	-1	6	-1	1	1	1	1	1
2	1	-1	-1	-1	-1	-1	6	1	-1	-1	-1	1	1
2	1	-1	1	1	1	1	6	1	-1	1	1	-1	-1
2	1	1	-1	1	1	-1	6	1	1	-1	1	-1	1
2	1	1	1	-1	-1	1	6	1	1	1	-1	1	-1
3	-1	-1	-1	1	1	1	7	-1	-1	-1	1	-1	-1
3	-1	-1	1	-1	-1	-1	7	-1	-1	1	-1	1	1
3	-1	1	-1	-1	-1	1	7	-1	1	-1	-1	1	-1
3	-1	1	1	1	1	-1	7	-1	1	1	1	-1	1
3	1	-1	-1	-1	1	-1	7	1	-1	-1	-1	-1	1
3	1	-1	1	1	-1	1	7	1	-1	1	1	1	-1
3	1	1	-1	1	-1	-1	7	1	1	-1	1	1	1
3	1	1	1	-1	1	1	7	1	1	1	-1	-1	-1
4	-1	-1	-1	-1	-1	-1	8	-1	-1	-1	-1	1	1
4	-1	-1	1	1	1	1	8	-1	-1	1	1	-1	-1
4	-1	1	-1	1	1	-1	8	-1	1	-1	1	-1	1
4	-1	1	1	-1	-1	1	8	-1	1	1	-1	1	-1
4	1	-1	-1	1	-1	1	8	1	-1	-1	1	1	-1
4	1	-1	1	-1	1	-1	8	1	-1	1	-1	-1	1
4	1	1	-1	-1	1	1	8	1	1	-1	-1	-1	-1
4	1	1	1	1	-1	-1	8	1	1	1	1	1	1

Alias Structure: Blk=CDEF Blk=ABEF Blk=BDF

Block*Factor Confounding Structure:

Blk*A	Blk*B	Blk*C	Blk*D	Blk*E	Blk*F
DE	CE	BE	BF	BC	BD
CF	DF	AF	AE	AD	AC

Design 7: 7 Factors in 4 Blocks of Size 16

The 64 treatment combinations are a one-half fraction.

Alias Structure: G = ACE Blk= CDEF Blk= ABEF

Same design matrix as for 6 Factors is 4 Blocks of Size 16 by defining the seventh column G as G=ACE.

Block*Factor Confounding Structure:

Blk*A	Blk*B	Blk*C	Blk*D	Blk*E	Blk*F	Blk*G

Note: No two factor interactions are confounded with Blk*X

Design 8: 7 Factors in 8 Blocks of Size 8

The 64 treatment combinations are a one-half fraction.

Alias Structure: G = ABCDEF Blk= CDEF Blk= ABEF Blk= BDF

Same design matrix as for 6 Factors is 8 Blocks of Size 8 by defining the seventh column G as G=ABCDEF.

Block*Factor Confounding Structure:

Blk*A	Blk*B	Blk*C	Blk*D	Blk*E	Blk*F	Blk*G
DE	CE	BE	AE	AD	AC	AB
BG	AG	DG	CG	FG	EG	CD
CF	DF	AF	BF	BC	BD	EF

Design 9: 8 Factors in 4 Blocks of Size 16

The 64 treatment combinations are a one-fourth, resolution IV fraction.

Alias Structure: G = ACE H = BCE

Blk= CDEF Blk= ABEF

Same design matrix as for 6 Factors is 4 Blocks of Size 16 by defining the seventh column G as G=ACE and the eighth column H as H=BCE.

Block*Factor Confounding Structure:

Blk*A	Blk*B	Blk*C	Blk*D	Blk*E	Blk*F	Blk*G	Blk*H

Note: No two factor interactions are confounded with Blk*X

Design 10: 8 Factors in 8 Blocks of Size 16

The 128 treatment combinations are a one-half fraction.

Alias Structure: H = ABCDEFG

Blk= BCFG Blk= ACEG Blk= DEFG

Block*Factor Confounding Structure:

Blk*A	Blk*B	Blk*C	Blk*D	Blk*E	Blk*F	Blk*G	Blk*H

Note: No two factor interactions are confounded with Blk*X